Ques 1 :

a). Optimized Algo (Using heap)

(6 marks)

```
int arr[ ] = {10,5,90,3,15,70,20} ;
int k = 4 ;

int n = sizeof(arr) / sizeof(int) ;

priority_queue<int, vector<int>, greater<int> > pq ;   → Create a min heap

for(int i = 0 ; i < k ; i++)          } Push first k elements of
      pq.push(arr[i]) ;               } array in heap

for(int i = k ; i < n ; i++)
{
      int se = pq.top() ;             → Find the smallest element in heap

      if(arr[i] > se)                 → check if current element of array
      {                                 is larger than the smallest element
            pq.pop() ;                  of heap. If yes, remove the smaller
            pq.push(arr[i]) ;           element and push the current
      }                                 element of array.
}

while(! pq.empty())                   → Finally we have k largest
{                                       elements in heap.
      cout << pq.top() << " " ;         Apply a loop - keep on printing
      pq.pop() ;                        the elements and removing them
}
```

Time Complexity : $O(n \log k)$

At any time only k elements will be present in heap, so addition and removal of 1 element will be of log k complexity.
We will add and remove n elements from a heap of size k, so overall complexity will be n log k.

Space Complexity : $O(k)$

We are maintaining an extra heap which contains k elements. So, space complexity will be $O(k)$

Ques 1: (4 Marks)

b).

		I_0	I_1	I_2	I_3	I_4	I_5	I_6
Profit	=	10	5	15	7	6	18	3
weight	=	2	3	5	7	1	4	1
$\frac{\text{Profit}}{\text{weight}}$	=	5	1.67	3	1	6	4.5	3

Sort in Dec Order

$I_4 \quad I_0 \quad I_5 \quad I_2 \quad I_6 \quad I_1 \quad I_3$

Item	Remaining Capacity	Profit
—	15	0
I_4	$15-1=14$	6
$I_4 \; I_0$	$14-2=12$	$6+10=16$
$I_4 \; I_0 \; I_5$	$12-4=8$	$16+18=34$
$I_4 \; I_0 \; I_5 \; I_2$	$8-5=3$	$34+15=49$
$I_4 \; I_0 \; I_5 \; I_2 \; I_6$	$3-1=2$	$49+3=52$
$I_4 \; I_0 \; I_5 \; I_2 \; I_6 \; I_1$	we can't put entire I_1 fraction = $\frac{\text{Remaining Capacity}}{\text{wt. of } I_1}$ $= \frac{2}{3}$	$52+\frac{2}{3}\times 5$ 55.33

Profit = 55.33

Ques 2:

a) (2 + 2 = 4 Marks)

i) NO
ii) Yes

} Give Example

b) (6 Marks)

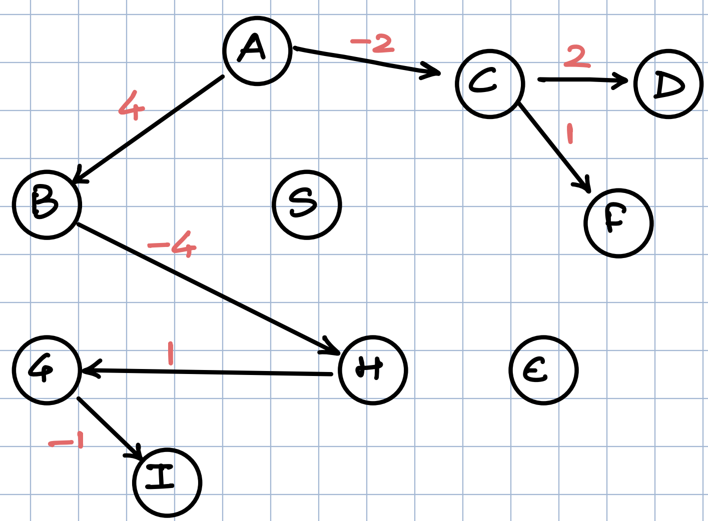
n = 10
relax every edge 9 times

Edges

A → C : -2
A → B : 4
C → D : 2
C → F : 1
S → A : 7
S → C : 6
S → F : 5
S → E : 6
E → F : -2
E → H : 3
B → G : -2
B → H : -4
H → G : 1
G → I : -1
I → H : 1
F → D : 3

Cost Initial	Relax 1^{st}	Relax 2^{nd}
A → 0	0	0
B → ∞	4	4
C → ∞	-2	-2
D → ∞	0	0
E → ∞	∞	∞
F → ∞	-1	-1
G → ∞	1	1
H → ∞	0	0
I → ∞	0	0
S → ∞	∞	∞

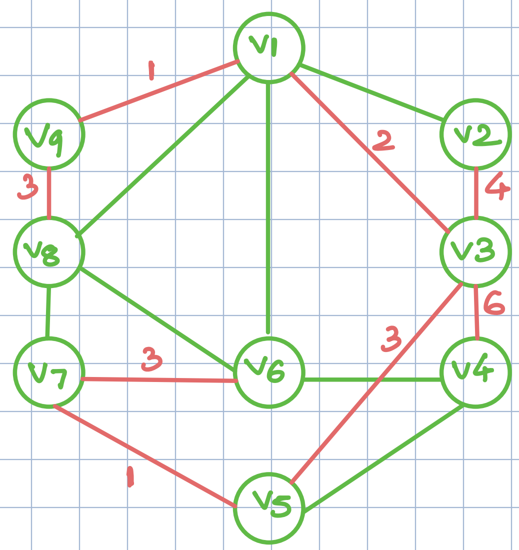
No change in weight after relaxing 2^{nd} time. Stop.



Ques 3:

(7 Marks)

a)

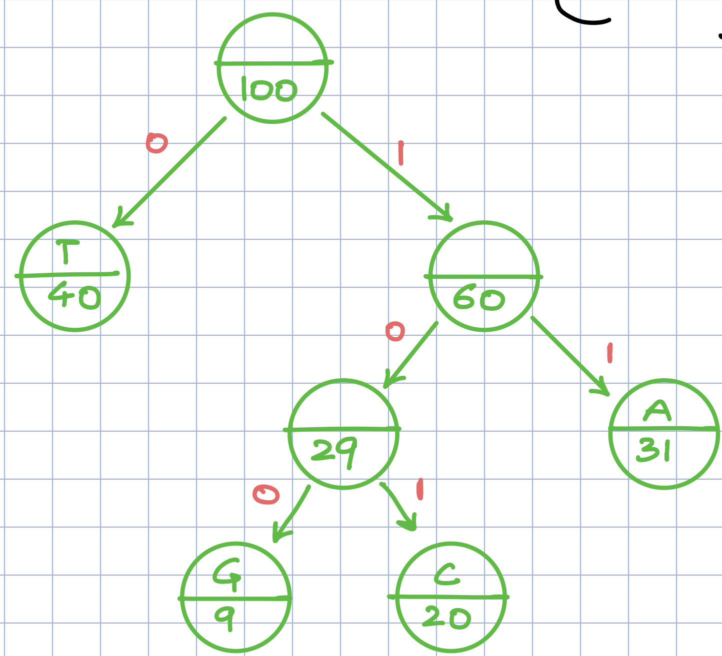


mst cost = 1+1+2+3+3+3+4+6
= 23

b)

	Frequency
A	31
C	20
G	9
T	40

(3 Marks)



T: 0
G: 100
C: 101
A: 11

Ques 4: (10 Marks)

10×5 $\quad$ 5×15 $\quad$ 15×8 $\quad$ 8×20

A $\quad$ B $\quad$ C $\quad$ D

no. of matrix operations required for multiplying 2 matrices:

$A \cdot B = 10 \times 5 \times 15 = 750$
$B \cdot C = 5 \times 15 \times 8 = 600$
$C \cdot D = 15 \times 8 \times 20 = 2400$

no. of matrix operations required for multiplying 3 matrices:

$A \cdot B \cdot C$

ABC

(A)(BC)

0 $\quad$ 600

10×5 $\quad$ 5×8

$10 \times 5 \times 8 = 400$

$0 + 600 + 400$
$= 1000$

(AB)(C)

750 $\quad$ 0

10×15 $\quad$ 15×8

$10 \times 15 \times 8 = 1200$

$750 + 0 + 1200$
$= 1950$

minimum multiplications needed for multiplying ABC = 1000

B·C·D

BCD

(B)(CD)

0 2400

5x15 15x20

5x15x20 = 1500

(BC)(D)

600 0

5x8 8x20

5x8x20 = 800

0+2400+1500
= 3900

600+0+800
= 1400

minimum multiplications needed for multiplying BCD = 1400

ABCD

ABCD

(A)(BCD)

0 1400

10x5 5x20

10x5x20 = 1000

(AB)(CD)

750 2400

10x15 15x20

10x15x20 = 3000

(ABC)(D)

1000 0

10x8 8x20

10x8x20 = 1600

0+1400+1000
= 2400

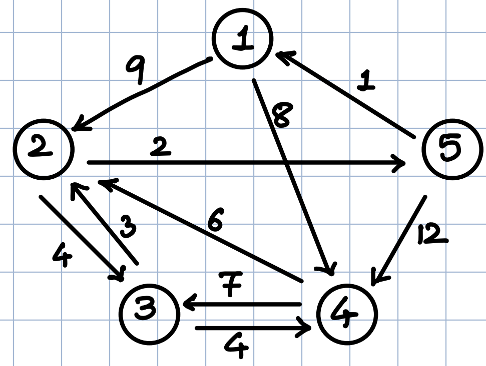
750+2400+3000
= 6150

1000+0+1600
= 2600

minimum multiplications needed for multiplying ABCD = 2400

Ques 5 : (10 Marks)

Considering wt $1 \to 4$ as 8



Original Matrix

$$\begin{array}{c|ccccc|c} & 1 & 2 & 3 & 4 & 5 & \\ \hline 1 & \infty & 9 & \infty & 8 & \infty & 8 \\ 2 & \infty & \infty & 4 & \infty & 2 & 2 \\ 3 & \infty & 3 & \infty & 4 & \infty & 3 \\ 4 & \infty & 6 & 7 & \infty & \infty & 6 \\ 5 & 1 & \infty & \infty & 12 & \infty & 1 \\ \hline & & & & & & 20 \end{array}$$

Row Reduction

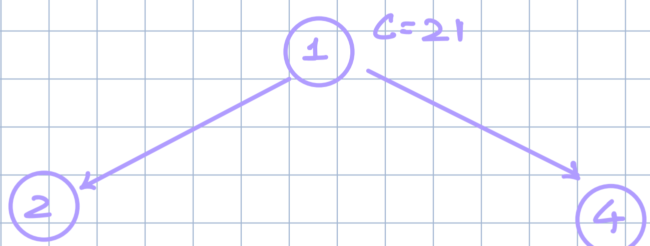
$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & 1 & \infty & 0 & \infty \\ 2 & \infty & \infty & 2 & \infty & 0 \\ 3 & \infty & 0 & \infty & 1 & \infty \\ 4 & \infty & 0 & 1 & \infty & \infty \\ 5 & 0 & \infty & \infty & 11 & \infty \\ \hline & & & 1 & & \end{array}$$

Col Reduction

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & 1 & \infty & 0 & \infty \\ 2 & \infty & \infty & 1 & \infty & 0 \\ 3 & \infty & 0 & \infty & 1 & \infty \\ 4 & \infty & 0 & 0 & \infty & \infty \\ 5 & 0 & \infty & \infty & 11 & \infty \end{array}$$

Reduced Matrix

Total Cost of Reduction = 20 + 1 = 21



$1 \to 2$

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & 1 & \infty & 0 & \infty \\ 2 & \infty & \infty & 1 & \infty & 0 \\ 3 & \infty & 0 & \infty & 1 & \infty \\ 4 & \infty & 0 & 0 & \infty & \infty \\ 5 & 0 & \infty & \infty & 11 & \infty \end{array}$$

Reduced Matrix

Make
1st row as ∞
2nd col as ∞
(2, 1) cell as ∞

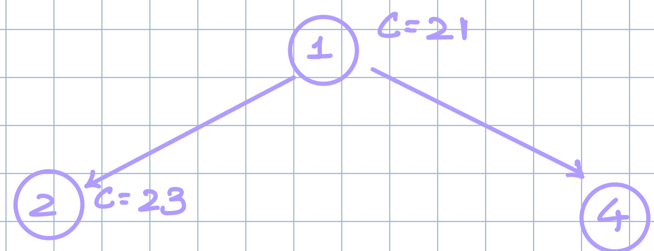
$$\begin{array}{c|ccccc|c} & 1 & 2 & 3 & 4 & 5 & \\ \hline 1 & \infty & \infty & \infty & \infty & \infty & \\ 2 & \infty & \infty & 1 & \infty & 0 & \\ 3 & \infty & \infty & \infty & 1 & \infty & 1 \\ 4 & \infty & \infty & 0 & \infty & \infty & \\ 5 & 0 & \infty & \infty & 11 & \infty & \end{array}$$

Row Reduction

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \infty & \infty & \infty & \infty & \infty \\
2 & \infty & \infty & 1 & \infty & 0 \\
3 & \infty & \infty & \infty & 0 & \infty \\
4 & \infty & \infty & 0 & \infty & \infty \\
5 & 0 & \infty & \infty & 11 & \infty
\end{array}
$$

Cost of Reduction = 1

$$
\begin{aligned}
\text{Cost}(1 \to 2) &= 21 + 1 + 1 \\
&= 23
\end{aligned}
$$



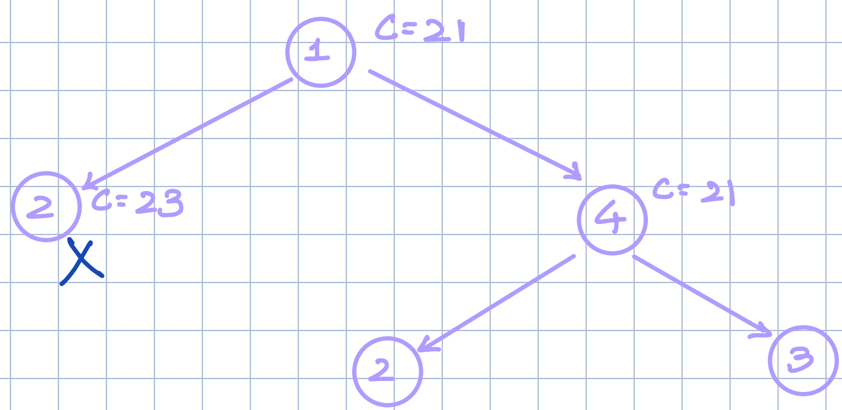
$1 \to 4$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \infty & 1 & \infty & 0 & \infty \\
2 & \infty & \infty & 1 & \infty & 0 \\
3 & \infty & 0 & \infty & 1 & \infty \\
4 & \infty & 0 & 0 & \infty & \infty \\
5 & 0 & \infty & \infty & 11 & \infty
\end{array}
\xrightarrow[\text{(4,1) cell as } \infty]{\substack{\text{Make} \\ \text{1st row as } \infty \\ \text{4th col as } \infty}}
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \infty & \infty & \infty & \infty & \infty \\
2 & \infty & \infty & 1 & \infty & 0 \\
3 & \infty & 0 & \infty & \infty & \infty \\
4 & \infty & 0 & 0 & \infty & \infty \\
5 & 0 & \infty & \infty & \infty & \infty
\end{array}
$$

Reduced Matrix

Cost of Reduction = 0

$$\text{Cost}(1 \to 4) = 21 + 0 + 0 = 21$$



$1 \to 4 \to 2$

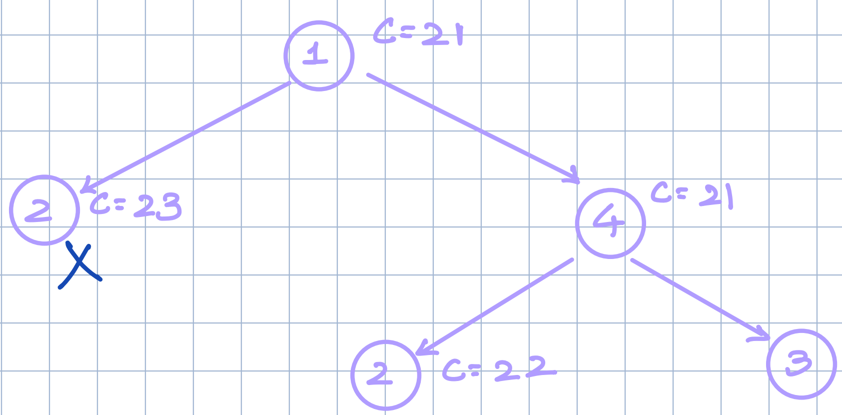
$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix} \end{array} \xrightarrow[\text{(2,4) cell as } \infty]{\text{Make 4th row as } \infty \text{, 2nd col as } \infty} \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix} \\ & \begin{array}{ccccc} & & 1 & & \end{array} \end{array}$$

Col Reduction ↓

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix} \end{array}$$

Cost of Reduction = 1

$$\text{Cost}(1 \to 4 \to 2) = 21 + 1 + 0 = 22$$

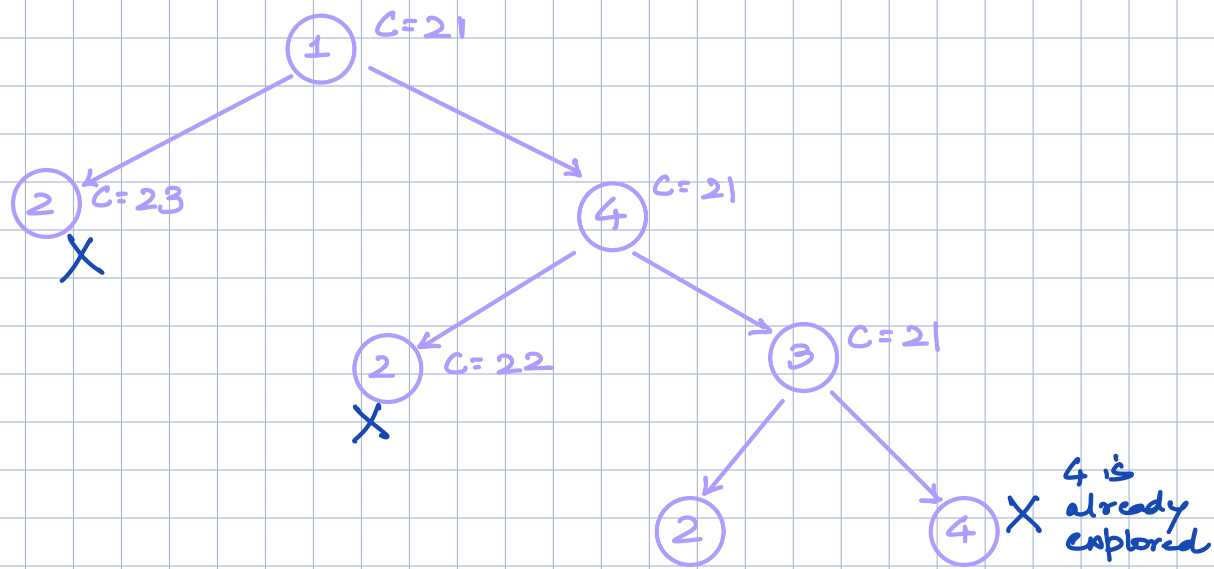


$1 \rightarrow 4 \rightarrow 3$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & 1 & \infty & 0 \\ 3 & \infty & 0 & \infty & \infty & \infty \\ 4 & \infty & 0 & 0 & \infty & \infty \\ 5 & 0 & \infty & \infty & \infty & \infty \end{array} \xrightarrow[\text{(3,4) cell as } \infty]{\substack{\text{Make} \\ \text{4th row as } \infty \\ \text{3rd col as } \infty}} \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & 0 \\ 3 & \infty & 0 & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & 0 & \infty & \infty & \infty & \infty \end{array}$$

Cost of Reduction = 0

$$\text{Cost}(1 \rightarrow 4 \rightarrow 3) = 21 + 0 + 0 = 21$$

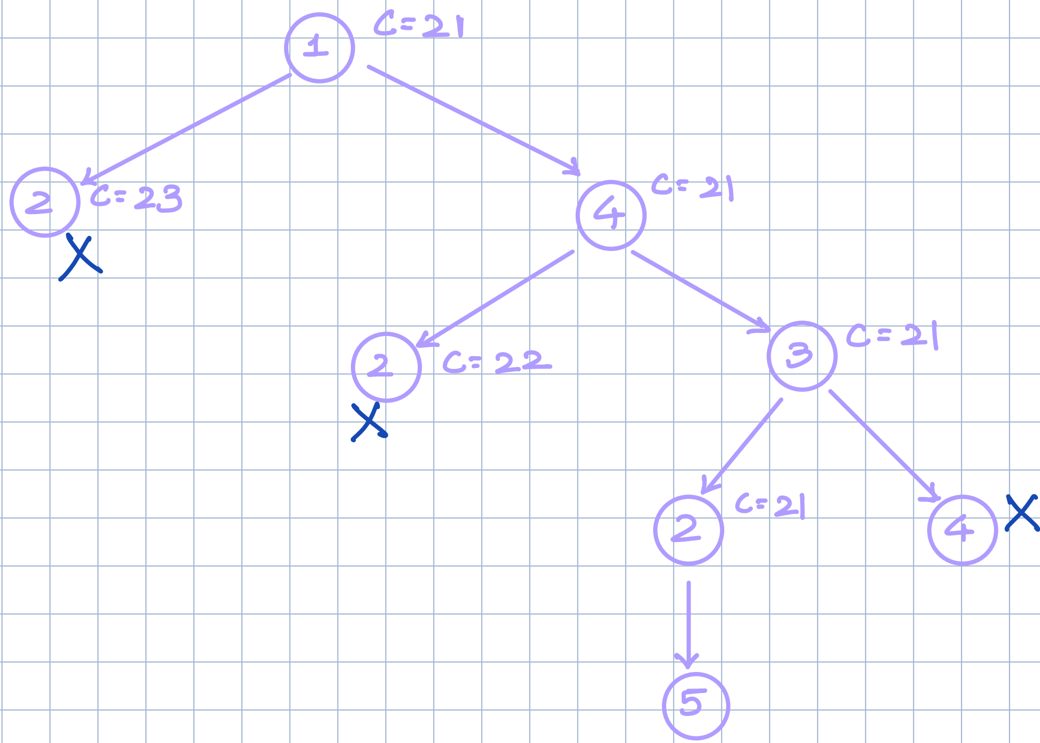


$1 \rightarrow 4 \rightarrow 3 \rightarrow 2$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & 0 \\ 3 & \infty & 0 & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & 0 & \infty & \infty & \infty & \infty \end{array} \xrightarrow[\text{(2,3) cell as } \infty]{\substack{\text{Make} \\ \text{3rd row as } \infty \\ \text{2nd col as } \infty}} \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & 0 \\ 3 & \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & 0 & \infty & \infty & \infty & \infty \end{array}$$

Cost of Reduction = 0

$$\text{Cost}(1 \rightarrow 4 \rightarrow 3 \rightarrow 2) = 21 + 0 + 0 = 21$$

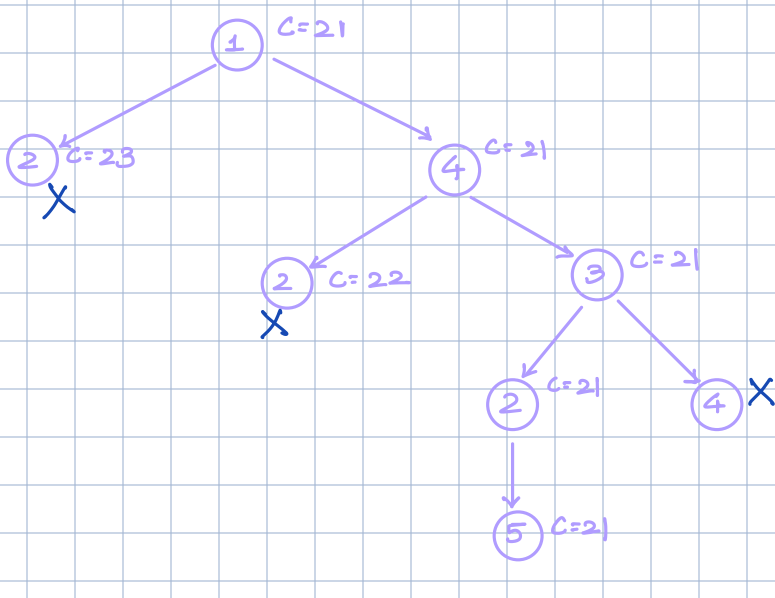


$1 \to 4 \to 3 \to 2 \to 5$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \infty & \infty & \infty & \infty & \infty \\
2 & \infty & \infty & \infty & \infty & 0 \\
3 & \infty & \infty & \infty & \infty & \infty \\
4 & \infty & \infty & \infty & \infty & \infty \\
5 & 0 & \infty & \infty & \infty & \infty
\end{array}
\xrightarrow[\text{(5,2) cell as } \infty]{\text{Make 2nd row as } \infty \text{, 5th col as } \infty}
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \infty & \infty & \infty & \infty & \infty \\
2 & \infty & \infty & \infty & \infty & \infty \\
3 & \infty & \infty & \infty & \infty & \infty \\
4 & \infty & \infty & \infty & \infty & \infty \\
5 & 0 & \infty & \infty & \infty & \infty
\end{array}
$$

Cost of Reduction = 0

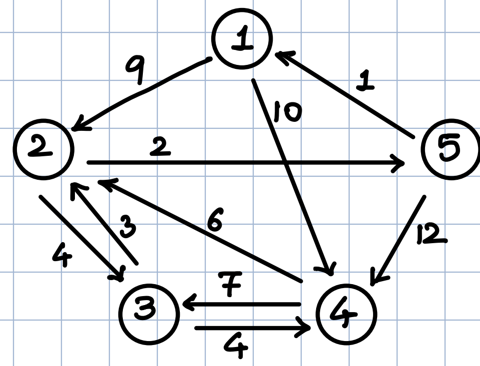
$\text{Cost}(1 \to 4 \to 3 \to 2 \to 5) = 21 + 0 + 0 = 21$



Path : $1 \to 4 \to 3 \to 2 \to 5 \to 1$
Cost : 21

Ques 5 :

Considering wt 1→4 as 10



Original Matrix

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 9 & \infty & 10 & \infty \\ \infty & \infty & 4 & \infty & 2 \\ \infty & 3 & \infty & 4 & \infty \\ \infty & 6 & 7 & \infty & \infty \\ 1 & \infty & \infty & 12 & \infty \end{bmatrix} \end{array} \quad \begin{array}{c} 9 \\ 2 \\ 3 \\ 6 \\ \underline{1} \\ 21 \end{array}$$

Row Reduction

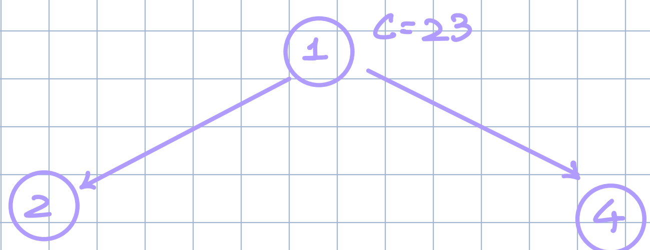
$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 0 & \infty & 1 & \infty \\ \infty & \infty & 2 & \infty & 0 \\ \infty & 0 & \infty & 1 & \infty \\ \infty & 0 & 1 & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \end{bmatrix} \\ & \begin{array}{ccccc} & & 1 & 1 & \end{array} \end{array}$$

Col Reduction

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 0 & \infty & 0 & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & 0 & \infty & 0 & \infty \\ \infty & 0 & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array}$$

Reduced Matrix

Total Cost of Reduction = 21 + 2 = 23



1→2

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 0 & \infty & 0 & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & 0 & \infty & 0 & \infty \\ \infty & 0 & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array}$$

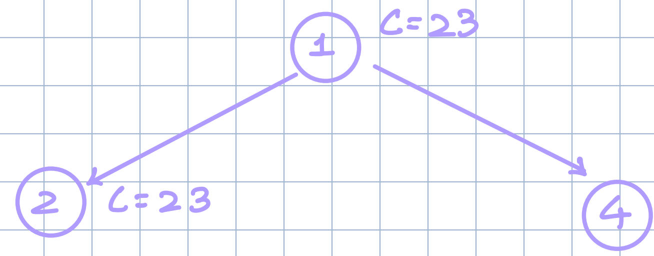
Reduced Matrix

Make
1st row as ∞
2nd col as ∞
(2, 1) cell as ∞

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array}$$

Cost of Reduction = 0

Cost $(1 \to 2) = 23 + 0 + 0 = 23$



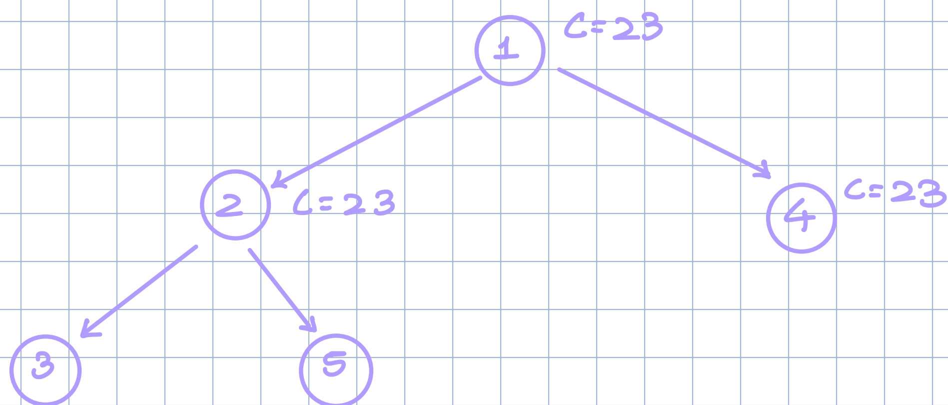
$1 \to 4$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \infty & 0 & \infty & 0 & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & 0 & \infty & 0 & \infty \\ \infty & 0 & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array} \xrightarrow[\text{(4,1) cell as } \infty]{\substack{\text{Make} \\ \text{1st row as } \infty \\ \text{4th col as } \infty}} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix} \end{array}$$

Reduced Matrix

Cost of Reduction = 0

Cost $(1 \to 4) = 23 + 0 + 0 = 23$



$1 \to 2 \to 3$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array} \xrightarrow[\text{(3,2) cell as } \infty]{\substack{\text{Make} \\ \text{2nd row as } \infty \\ \text{3rd col as } \infty}} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array}$$

Cost of Reduction = 0

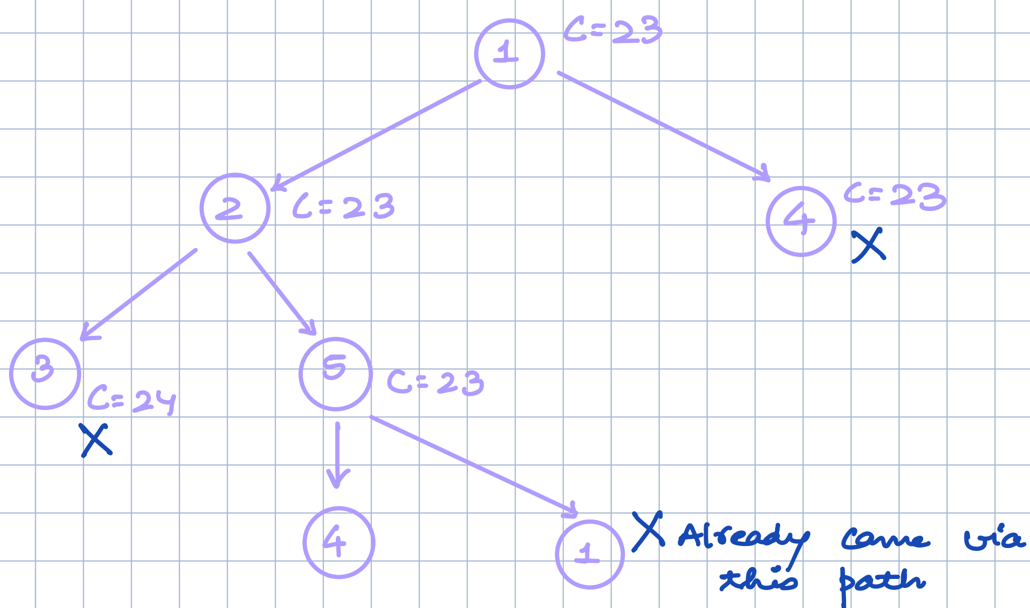
$$\text{Cost}(1 \to 2 \to 3) = 23 + 0 + 1 = 24$$

$1 \to 2 \to 5$

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & \infty & 0 \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array} \xrightarrow[(5,2)\text{ cell as } \infty]{\substack{\text{Make} \\ \text{2nd row as } \infty \\ \text{5th col as } \infty}} \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array}$$

Cost of Reduction : 0

$$\text{Cost}(1 \to 2 \to 5) = 23 + 0 + 0 = 23$$

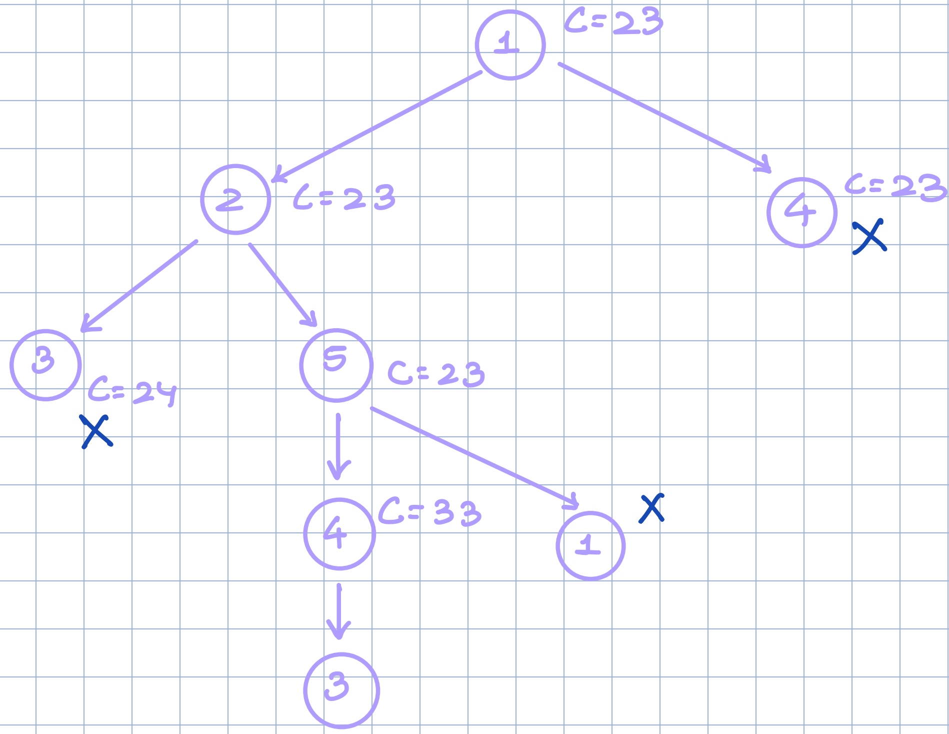


$1 \to 2 \to 5 \to 4$

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & 0 & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \end{bmatrix} \end{array} \xrightarrow[(4,5)\text{ cell as } \infty]{\substack{\text{Make} \\ \text{5th row as } \infty \\ \text{4th col as } \infty}} \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \end{array}$$

Cost of Reduction : 0

$$\text{Cost}(1 \to 2 \to 5 \to 4) = 23 + 0 + 10 = 33$$

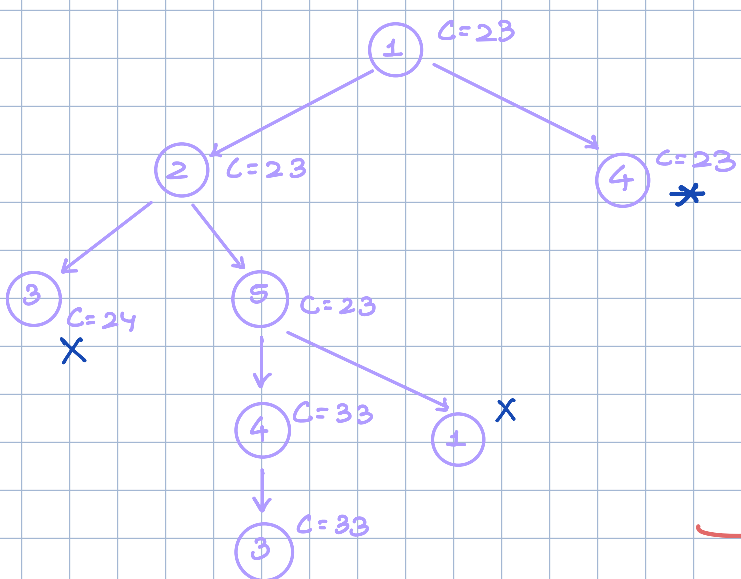


$1 \to 2 \to 5 \to 4 \to 3$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & 0 & \infty & \infty \\ 5 & \infty & \infty & \infty & \infty & \infty \end{array} \xrightarrow[\text{(3,4) cell as } \infty]{\text{Make 4th row as } \infty \text{, 3rd col as } \infty} \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & \infty & \infty & \infty & \infty & \infty \end{array}$$

Cost of Reduction = 0

Cost $(1 \to 2 \to 5 \to 4 \to 3)$ = $33 + 0 + 0 = 33$



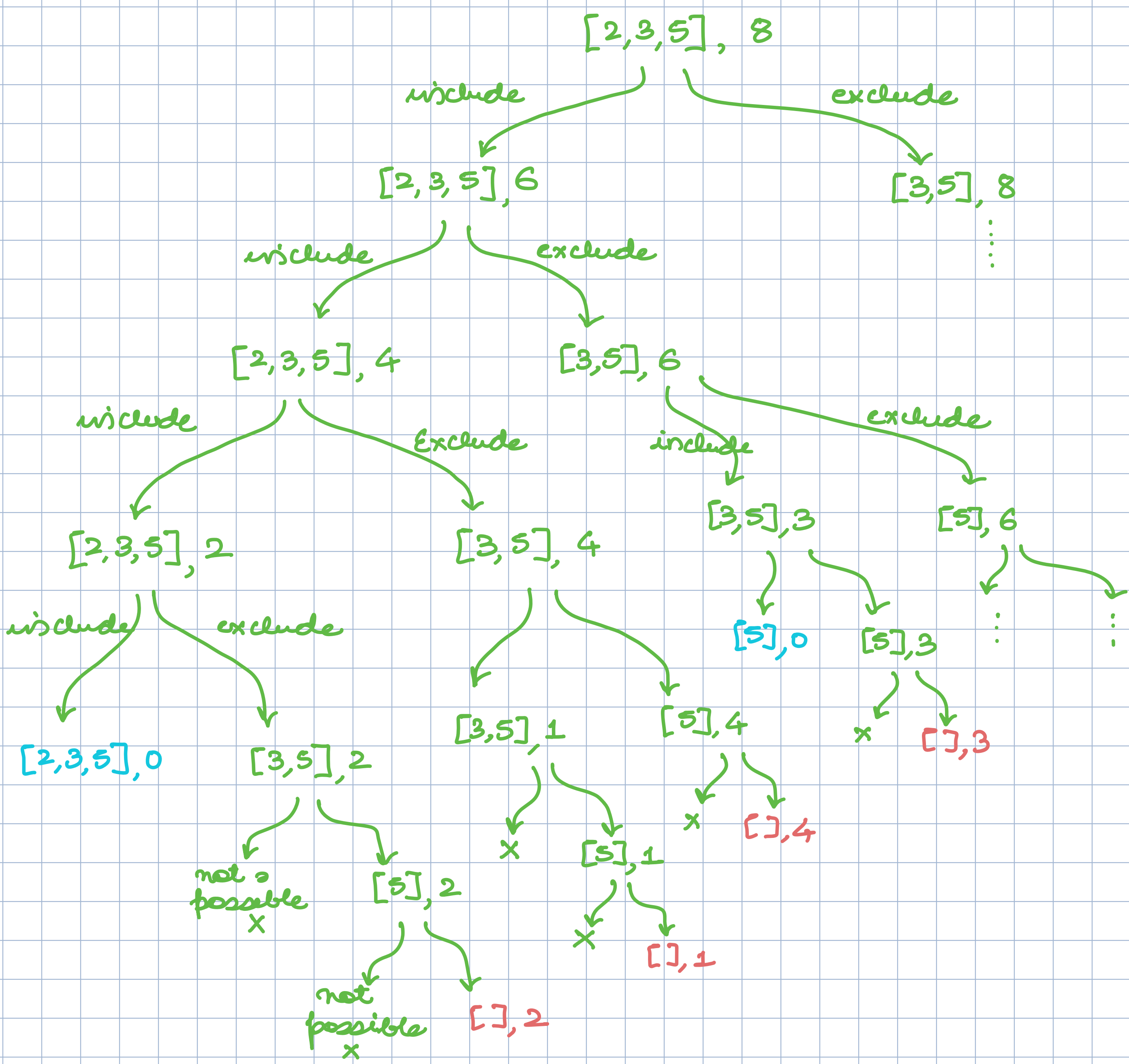
But no path to go from $3 \to 1$.
Instead of choosing $1 \to 2$, we should have chosen $1 \to 4$

Path : $1 \to 4 \to 3 \to 2 \to 5 \to 1$
Cost : $10 + 7 + 3 + 2 + 1$
: 23

Ques 6:

a).

(8 Marks)



Logic: Give 2 recursive calls : one for including the element and other for excluding the element.
Keep on storing the result in vector and when we hit the base case simply print it.

for virtually breaking the array

```
void sequences(int denom[ ], int n, int idx, int amount, vector <int> &ans)
{
    if(amount == 0)
    {
        for(int i : ans)
            cout << i << " " ;
        cout << endl ;
        return ;
    }

    if(idx == n)
        return ;

    // include
    if(amount >= denom[idx])
    {
        ans.push_back(denom[idx]) ;
        sequences(denom,n, idx,amount-denom[idx], ans) ;
        ans.pop_back() ;
    }

    // exclude
    sequences(denom,n, idx+1, amount, ans) ;

}

int main()
{
    int denom[ ] = {2,3,5} ;
    vector<int> ans ;
    int n = sizeof(denom) / sizeof(int) ;
    sequences(denom, n, 0, 8, ans) ;
    return 0 ;
}
```

vector to store answers

base case:
Check the amount if its 0
then print the vector.

if array is over then return

Include call

Backtracking step.
Remove the element

Exclude call

Time Complexity: Exponential – $O(2^k)$

k = ht of recursion tree

Space Complexity: Recursion extra space – $O(2^k)$

b) (2 Marks)

Optimal substructure:

Any optimal solution to a problem of size n is based on optimal solution to the same problem when considering <n elements

Overlapping Subproblems:

Problem is broken down into subproblems which are reused several times.